Session 5 : solutions

Exercise 1 : will be given in due time...

Exercise 2 :

$$c(\vec{r}) = \frac{1}{(2\pi)^3} \int d\vec{k} \; e^{i\vec{k}\cdot\vec{r}} \frac{1}{k^2 + k_0^2} =$$

$$= \frac{1}{(2\pi)^3} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dk \; k^2 \frac{1}{k^2 + k_0^2} e^{ikr\cos\theta} =$$

$$= \frac{1}{(2\pi)^2} \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} dk \; k^2 \frac{1}{k^2 + k_0^2} e^{ikr\cos\theta} =$$

$$= \frac{1}{(2\pi)^2} \int_0^{\infty} dk \frac{k^2}{k^2 + k_0^2} \int_0^{\pi} d\theta \sin\theta \; e^{ikr\cos\theta} =$$

$$= \frac{1}{(2\pi)^2} \int_0^{\infty} dk \frac{k^2}{k^2 + k_0^2} \left[-\frac{1}{ikr} e^{ikr\cos\theta} \Big|_0^{\pi} \right] =$$

$$= \frac{2}{(2\pi)^2} \int_0^{\infty} dk \frac{k^2}{k^2 + k_0^2} \frac{\sin(kr)}{kr} = \frac{2}{(2\pi)^2 r} \int_0^{\infty} dk \frac{k \sin(kr)}{k^2 + k_0^2} =$$

close in the upper plane — lower plane

$$= \frac{1}{(2\pi)^2 r} \int_{-\infty}^{+\infty} dk \, \frac{k \sin(kr)}{k^2 + k_0^2} = \frac{1}{(2\pi)^2 r} \int_{-\infty}^{+\infty} dk \, \frac{k}{(k+ik_0)(k-ik_0)} \left[\frac{e^{ikr} - e^{-ikr}}{2i} \right] =$$

use the even-ness of the integrand

from clockwise integration

residues

$$= \frac{2\pi \cancel{i}}{(2\pi)^2 r} \left\{ \frac{\cancel{i k_0}}{2\cancel{i k_0}} \, \frac{e^{-k_0 r}}{2\cancel{i}} - (-1) \, \frac{\cancel{-i k_0}}{\cancel{-2i k_0}} \, \frac{e^{-k_0 r}}{2\cancel{i}} \right\} =$$

$$= \frac{1}{4\pi r} \, e^{-k_0 r}$$